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IACPE No 19, Jalan Bilal Mahmood 80100 Johor Bahru Malaysia	SOLID AND FLUID MECHANICS CPE LEVEL I TRAINING MODULE	

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Scope

Solid mechanics is the study of the deformation and motion of solid materials under the action of forces, temperature changes, phase changes, and other external or internal agents. It is one of the fundamental applied engineering sciences in the sense that it is used to describe, explain and predict many of the physical phenomena.

Solid mechanics is typically useful in designing and evaluating tools, machines, and structures, ranging from wrenches to cars to spacecraft. The required educational background for these includes courses in statics, dynamics, and related subjects. For example, dynamics of rigid bodies is needed in generalizing the spectrum of service loads on a car, which is essential in defining the vehicle's deformations and long-term durability.

Fluid mechanics deals with the study of all fluids under static and dynamic situations. Fluid mechanics is a branch of continuous mechanics which deals with a relationship between forces, motions, and statistical conditions in a continuous material. This study area deals with many and diversified problems such as fluid statics, flow in enclose bodies, or flow round bodies (solid or otherwise), flow stability, etc.

Both solid mechanics and fluid mechanics play very important roles in design. Because a fluid cannot resist deformation force, it moves, or flows under the action of the force. Its shape will change continuously as long as the force is applied. Whereas, a solid can resist a deformation force while at rest. While a force may cause some displacement, the solid does not move indefinitely.

Solid mechanics which is based on Newton laws, either in rest or motion. Solid mechanics consist of several fundamentals such as vectors, moments, couple, moment inertia, motion, vibration, and rigid bodies in statics and dynamics. Fluid mechanics consist of fluid properties and hydrostatic forces along with their application in nature.

INTRODUCTION

General Consideration

A. Aspects of Solid Mechanics

The theory of solid mechanics starts with the particle, and then a rigid body. Rigid body mechanics is usually subdivided into statics and dynamics.

Statics

Statics are the study of materials at rest. The actions of all external forces acting on such materials are exactly counterbalanced and there is a zero net force effect on the material: such materials are said to be in a state of static equilibrium. Equilibrium is said to be stable when the body with the forces acting upon it returns to its original position after being displaced a very small amount from that position; and neutral when the forces retain their equilibrium when the body is in its new position.

If a body is supported by other bodies while subject to the action of forces, deformations and forces will be produced at the points of support or contact and these internal forces will be distributed throughout the body until equilibrium exists. They are equal in magnitude and opposite in direction to the forces with which the supports act on the body, known as supporting forces. The supporting forces are external forces applied to the body[6].

A material body can be considered to consist of a very large number of particles. A rigid body is one which does not deform, in other words, the distance between the individual particles making up the rigid body remains unchanged under the action of external forces. An example of the statics of a rigid body is a bridge supporting the weight a car.

Dynamics

There are two major categories in dynamics, kinematics and kinetics. Kinematics involves the time and geometry-dependent motion of a particle, rigid body, deformable body, or a fluid without considering the forces that cause the motion. It relates position, velocity, acceleration, and time. Kinetics combines the concepts of kinematics and the forces that cause the motion.

Dynamics of a rigid body follows Newton's second law. Newton's second law of motion states that the body will accelerate in the direction of and proportional to the magnitude of the resultant R. In rectilinear motion, the acceleration and the direction of the unbalanced force must be in the direction of motion. Forces must be in balance and the acceleration equal to zero in any direction other than the direction of motion. An example of the dynamics of rigid body is an accelerating and decelerating elevator.

General Laws

The fundamental concepts and principles of mechanics follow the relation between the motion and the force that is defined by Newton's Law. Newton's law states that:

1. A body remains at rest or continues in a straight line at a constant velocity unless acted upon by an external force.
2. A force applied to a body accelerates the body by an amount which is proportional to the force.
3. Every action is opposed by an equal and opposite reaction.

B. Aspect of Fluid Mechanics

Fluid mechanics is a study of the relationships between the effects of forces, energy and momentum occurring in and around a fluid system. Fluids are substances capable of following and taking the shape of containers. Fluids can be classified as liquids or gases; liquids are incompressible, occupy definite volumes, and have free surfaces; whereas, gases are compressible and expand until they occupy all portions of the container. Fluids cannot sustain shear or tangential forces when in equilibrium.

Substances may be classified by their response when at rest to the imposition of a shear force. Liquid that undergoes a shear stress between a short distance of two plates can be shown in Figure 1.

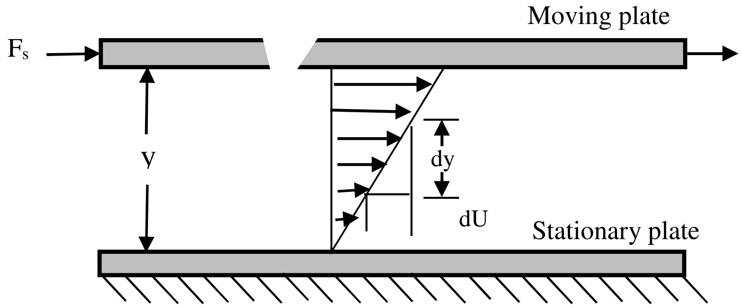


Figure 1 : schematics to describe the shear stress in fluid mechanics

The deformation characteristics of various substances are divided to five characteristics as illustrated in figure 2. An ideal or elastic solid will resist the shear force, and its rate of deformation will be zero regardless of loading and hence is coincident with the ordinate. A plastic will resist the shear until its yield stress is attained, and the application of additional loading will cause it to deform continuously, or flow. If the deformation rate is directly proportional to the flow, it is called an ideal plastic.

A Newtonian fluid is a real fluid which has internal friction so that their rate of deformation is proportional to the applied shear stress. If it is not directly proportional, it is called a non-Newtonian fluid.

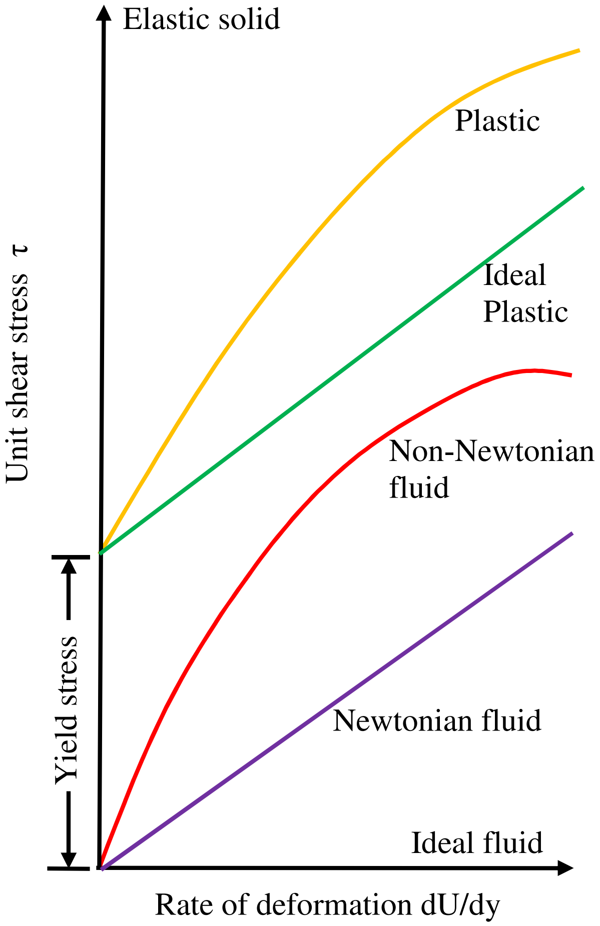


Figure 2 : deformation characteristics of substances

Fluid Statics

In a static fluid, an important property is the pressure in the fluid. Pressure is defined as force exerted by a mass under the influence of gravity and a mass of fluid on a supporting area, or force per area. The fluid pressure acts normal to any plane and is transmitted with equal intensity in all directions. In fluid mechanics and in thermodynamic equations, the units are lbf/ft^2, but engineering practice is to use units of lbf/in^2.

Most fluid-mechanics equations and all thermodynamic equations require the use of absolute pressure, and unless otherwise designated, a pressure should be understood to be absolute pressure. Common practice is to denote absolute pressure as lbf/ft^2 abs, or psfa, lbf/in^2 abs or psia; and in a like manner for gauge pressure lbf/ft^2 g, lbf/in^2 g, and psig. The relationship between absolute pressure, gauge pressure, and vacuum is shown in Figure 3 [6].

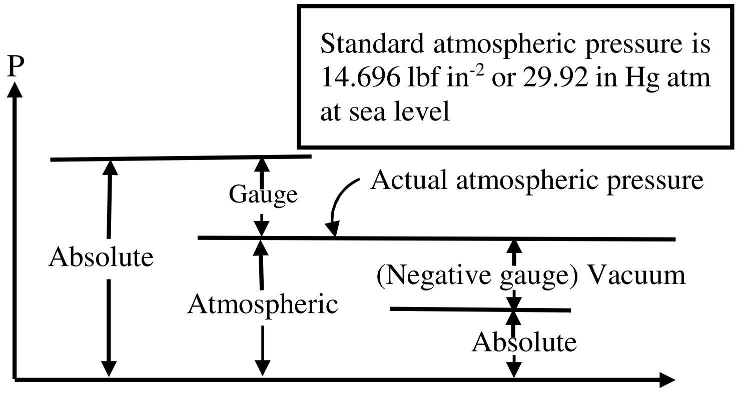


Figure 3 : pressure relation

Fluid Dynamics

Fluid dynamics is a sub-discipline of fluid mechanics that deals with fluid flow – natural science of fluid in motion. The elements of a flowing fluid can move at different velocities and can be subjected to different accelerations. The following principles apply in fluid flow:

a. The principle of conservation of mass, from which the equation of continuity is developed.
b. The principle of kinetic energy, from which some flow equations are derived.
c. The principle of momentum, from which equations regarding the dynamic forces exerted by flowing fluids can be established.

Types of Fluid

Fluid flow can be characterized as steady or unsteady, uniform or non-uniform. There typically can classify any flow as follow.

1. Steady uniform flow

   Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.

2. Steady non-uniform flow

   Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as moving along the length of the pipe toward the exit.

3. Unsteady uniform flow

   At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.

4. Unsteady non-uniform flow

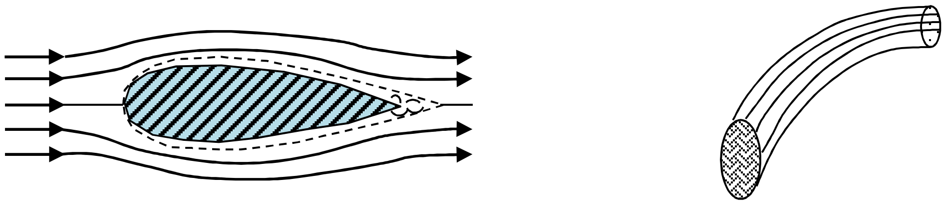
   Every condition of the flow may change from point to point and with time at every point. For example waves in a channel.

In additional, there are also considered about number of dimensional required to describe the velocity profile.

1. Flow is one dimensional, if the flow parameters (such as velocity, pressure, depth etc.) at a given instant in time only vary in the direction of flow and not across the cross-section.
2. Flow is two-dimensional, if it can be assumed that the flow parameters vary in the direction of flow and in one direction at right angles to this direction. Streamlines in two-dimensional flow are curved lines on a plane and are the same on all parallel planes. The example is area.
3. All flows take place between boundaries that are three-dimensional. Typically, it takes three directions of flow such as x, y, z as a volume of fluid in a circular pipe.

Streamlines and Stream-tube

A streamline is a line which gives the direction of the velocity of a fluid particle at each point in the flow stream. When streamlines are connected by a closed curve in steady flow, they will form a boundary through which the fluid particles cannot pass. The space between the streamlines becomes a stream tube. They can be illustrated in figure 4.



(a) streamlines around a wing (b) stream-tube

Figure 4 : streamlines and stream-tube

The Laws of Thermodynamics

The First Law of Thermodynamics

This is a law of energy conservation. It states that energy is always conserved, it cannot be created or destroyed. In essence, energy can be converted from one form into another[9]. The increase in internal energy of a closed system can be defined as follows.

$$Q - W = \Delta U \qquad \text{Eq (1)}$$

Where,

Q = heat transfer, Btu (kJ)
W = work transfer, Btu (kJ)
ΔU = increase in internal energy, Btu (kJ)

For a thermodynamic cycle of a closed system, which returns to its original state, the heat Q_{in} supplied to a closed system in one stage of the cycle, minus that Q_{out} removed from it in another stage of the cycle, equals the net work done by the system. Work done by a system is considered positive; $W > 0$. Work done on a system is considered negative; $W < 0$.

The Second Law of Thermodynamics

It might be thought that the first law of thermodynamics permits all the heat transfer to a cycle to be returned as work transfer, but unfortunately the second law places restraints on the achievement of this desirable situation. It states that in all energy exchanges, if no energy enters or leaves the system, the potential energy of the state will always be less than that of the initial state. This is also commonly referred to as entropy. Entropy is a measure of disorder[13].

DEFINITION

Angular displacement - a particle moves on a circular path its angle of rotation (or its angular displacement) θ, varies with time.

Boundary layer - thin layer of fluid adjacent to a surface where viscous effects are important; viscous effects are negligible outside the boundary layer.

Buoyancy - is based on Archimedes' principle, which states that the buoyant force exerted on a submerged body is equal to the weight of the displaced fluid.

Center of gravity - the point through which the whole weight of a body may be assumed to act.

Couple - pair of two equal and opposite forces acting on a body in a such a way that the lines of action of the two forces are not in the same straight line.

Drag coefficient - force in the flow direction exerted on an object by the fluid flowing around it, divided by dynamic pressure and area.

Energy - the capacity of a body to do work by reason of its motion or configuration.

Entropy - a measure of the disorder of any system, or of the unavailability of its heat energy for work.

Force - the action of one body on another which will cause acceleration of the second body unless acted on by an equal and opposite action counteracting the effect of the first body.

Friction - the resistance that is encountered when two solid surfaces slide or tend to slide over each other.

Impulse - the product of the force and the time that force acts

Inertia - property of matter which causes a resistance to any change in the motion of a body.

Isentropic - one condition for which there is no heat transfer in reversible between the system and surroundings, therefore this process is also adiabatic.

Lift coefficient - force perpendicular to the flow direction exerted on an object by the fluid flowing around it, divided by dynamic pressure and area.

Linear momentum - the product of mass and the linear velocity of a particle and is a vector.

Metacenter - the point at which the line of action of the buoyancy force cuts the vertical center line of the floating body in the displaced position.

Moment of the force - the turning effect of a force on a body.

Polar moment of inertia - the sum of the moments of inertia about any two axes at right angles to each other in the plane of the area and intersecting at the pole.

Potential energy - the energy possessed by an element of fluid due to its elevation above a reference datum.

Rigid body - one in which the particles are rigidly connected that does not deform, or change shape.

Resonance - characteristic through increasing amplitude to infinity. The resonance phenomenon appears when the frequency of perturbation or forced angular frequency, p is equal to the natural angular frequency ω.

Steady uniform flow - conditions do not change with position in the stream or with time.

Streamline - a line which gives the direction of the velocity of a fluid particle at each point in the flow stream.

Separation - phenomenon that occurs when fluid layers adjacent to a solid surface are brought to rest and boundary layers depart from the surface contour, forming a low pressure wake region. Separation can occur only in an adverse pressure gradient.

Transition - change from laminar to turbulent flow within the boundary layer.

Vector - a directed line segment that has both magnitude and direction.

Viscosity - a measure of the resistance fluids to flow and may be considered as internal friction.

NOMENCLATURE

A	sectional area, ft^2 (m^2)
a	total angular acceleration, rad/s^2
c	actual damping coefficient, N.s/m
c_{cr}	critical damping coefficient, N.s/m
C_D	drag coefficient, dimensionless
Cv	constant volume heat capacity, Btu/lbmol.^{o}F (J/mol.K)
d	distance, ft (m)
dv	velocity differential, ft/s (m/s)
dy	distance differential, ft (m)
f	frequency, Heartz (rad/s)
F	force, lbf (N)
F_B	buoyant force, lbf (N)
F_D	drag force, lbf (N)
F_H	normal force on the vertical projection, lbf (N)
F_0	amplitude of the forced vibration, dimensionless
F_V	weight of fluid above the curve, lbf (N)
g	acceleration gravitional, 32.2 ft/s^2 (9.81 m/s^2)
h	height above the ground, ft (m)
H	head, ft (m)
H_1	enthalpy at point 1, btu/lb (J/kg)
H_2	enthalpy at point 2, btu/lb (J/kg)
h_A	head added, ft (m)
h_L	head loss, ft (m)
h_E	head extracted, ft (m)
H_O	angular momentum about O, (kg.m^2/s)
I	moment of inertia, lbm.ft^2 (kg.m^2)
k	radii of gyration, ft (m)
k	spring stiffness, N/m
KE	kinetic energy, J (N.m)
m	mass, lb (kg)
M	moment, lbf.ft (Nm)
MF_i	mole fraction of component *i*

MW_i	molecular weight of component *i*, lb/ft^3 (kg/m^3)
P	power, HP (watt)
PE	potential energy, J (N.m)
Q	energy added, Btu/lb (J/kg)
r	radius of circular path, ft (m)
R	gas law constant, 10.731 ft^3.lb$_f$/in^2.lbmol °R (8314.34 kg.m^2/s^2.kgmol.K)
s	space or displacement, ft (m)
S	entropy, btu/lbm.°F (kJ/kg.K)
T	period, sec/cycle
T	temperature, °R (K)
U	internal energy, btu (J)
v	angular velocity, rad/s
v	velocity, ft/s (m/s)
V	volume, ft^3 (m^3)
W	weight, lbf (N)
W	work, J (N.m)
Ws	net mechanical work, Btu/lb (J/kg)
z	elevation or depth, ft (m)

Greek letters

$\dot{m}$	mass flow rate, lbm/s (kg/s)
α	angle of inclination, degree
α	angular acceleration, rad/sec^2
α	kinetic energy velocity correction factor
a_n	normal acceleration, rad/s^2
a_t	tangential acceleration, rad/s^2
β	momentum velocity correction factor
ε	absolute roughness, in (mm)
$\sum F$	total of force, lbf (N)
$\sum F$	frictional losses, Btu/lb (J/kg)
δ	polytropic index
θ	angular displacement, rad
ω	angular velocity, rad/s
ω_d	damped natural frequency, Hz (rad/s)
ζ	damping factor, ratio c/c_{cr}

ρ	radius of curvature of the path, ft (m)
τ	shear force, lb/ft.s^2 (N/m^2)
μ	viscosity, lbf.s/ft^2 or poise (N.s/m^2)
μ_m	gas mixture viscosity, micropoise (N.s/m^2)
ν	kinematic viscosity, ft^2/s or stoke (m^2/s)
γ	specific weight, lbf/ft^3 (N/m^3)
γ	specific heat ratio, Cp/Cv

THEORY

A. Solid Mechanics

I. Fundamental of Solid Mechanic Statics

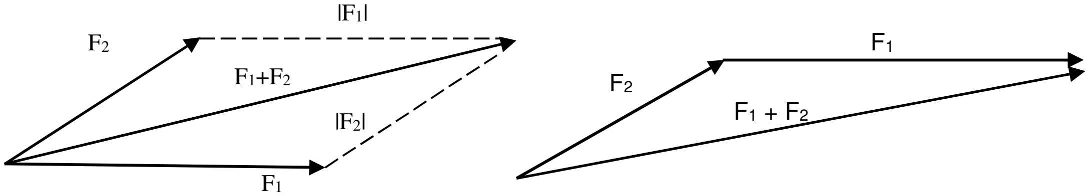
Vectors

A fundamental that should be mastered in statics of a rigid body is the vector. Two kinds of quantities are used in engineering mechanics. A scalar quantity has only magnitude (mass, time, temperature, etc.). A vector quantity has magnitude and direction (force, velocity, etc.). Vectors are represented here by arrows and are used in analysis according to universally applicable rules that facilitate calculations in a variety of problems[10].

Characteristics of Vector

a. Vector Addition

Any number of concurrent vectors may be summed, mathematically or graphically, and in any order. The vectors F_1 and F_2 add according to the parallelogram law: $F_1 + F_2$ is equal to the diagonal of a parallelogram formed by the graphical representation of the vectors as shown in figure 5(a). The vectors also can be added by moving them successively to parallel positions so that the head of one vector connects to the tail of the next vector as shown in figure 5(b).



(a) (b)

Figure 5: vectors addition

b. Resolution of Vectors and Components

A resultant force may be resolved into two forces at right angles to another. The resultant shown is at angle θ to *x* axis as follow.

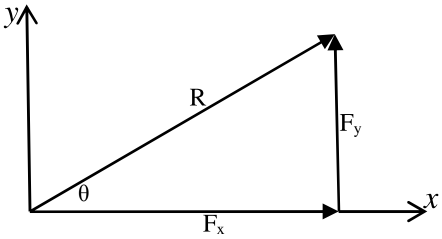


Figure 6: Triangle of forces

There are components and resultant which can be defined as follow.

$F_x = R\cos\theta$ Eq (2)
$F_Y = R\sin\theta$ Eq (3)

Therefore, the angle and resultant can be obtained by the components as below:

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) \qquad \text{Eq (4)}$$

$$R = \sqrt{F_x^2 + F_y^2} \qquad \text{Eq (5)}$$

c. Scalar Product of Two Vectors

The scalar (dot) product of two concurrent vectors A and B is defined by

$$a \cdot b = b \cdot a = |a||b|\cos\theta \qquad \text{Eq (6)}$$

where A and B are the magnitudes of the vectors, they are defined as follow.

$$A \cdot B = B \cdot A = A_xB_x + A_yB_y + A_zB_z \qquad \text{Eq (7)}$$

$$\theta = \arccos \frac{A_xB_x + A_yB_y + A_zB_z}{AB} \qquad \text{Eq (8)}$$

For the scalar triple product, this scalar product is used in calculating moments. A, B, C can be expressed in the following determinant form:

$$A \cdot (B \times C) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = A_x\left(B_yC_z - B_zC_y\right) + A_y\left(B_zC_x - B_xC_z\right) + A_z\left(B_xC_y - B_yC_x\right) \qquad \text{Eq (9)}$$

d. Vector Product of Two Vectors

A powerful method of vector mechanics is available for solving complex problems, such as the moment of a force in three dimensions. The vector (cross) product of a vector A and a vector B is defined as A x B that should be perpendicular to the plane of vectors A and B. There are several rules :

The sense of the unit vector n that appears in the definition of A x B depends on the order of the factors A and B in such a way that

$$A \times B = -(B \times A) \qquad \text{Eq (10)}$$

The magnitude of a x b is given by

$$|A \times B| = |A||B| \sin\theta \qquad \text{Eq (11)}$$

A set of mutually perpendicular unit cordinate vectors i, j, k is called right-handed when i x j = k and left-handed when i x j = -k.

The vector product is calculated using a determinant form as follows.

$$A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = A_y B_z i + A_z B_x j + A_x B_y k - A_y B_x k - A_x B_z j - A_z B_y i \qquad \text{Eq (12)}$$

For three vectors, the vector triple product of three vectors A, B, C is the vector A x(B x C) as defined by

$$A \times (B \times C) = A \cdot CB - A \cdot BC \qquad \text{Eq (13)}$$

Statics of Rigid Bodies

All solid materials deform when forces are applied to them, but often it is reasonable to model components and structures as rigid bodies, at least in the early part of the analysis. The forces on a rigid body are generally not concurrent at the center of mass of the body, which cannot be modeled as a particle if the force system tends to cause a rotation of the body. Resultant of forces acting on a body can be considered by number of forces, which can be described as follow.

a. Resultant of Forces Acting on a Body at the Same Point

The resultant R of two forces F_1 and F_2 applied to a rigid body at the same point is represented in magnitude and direction by the diagonal of the parallelogram formed by F_1 and F_2. It can be illustrated in figure 7.

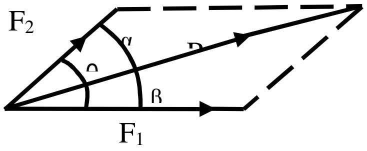


Figure 7: resultant of two forces

The resultant and degrees between the result are defined by

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} \qquad \text{Eq (14)}$$

And,

$$\text{Sin } \alpha = (F_1 \sin\theta) / R \qquad \text{Eq (15)}$$

$$\text{Sin } \beta = (F_2 \sin\theta) / R \qquad \text{Eq (16)}$$

b. Lami's Theorem

If three coplanar forces acting on a point in a body keep it in equilibrium, then each force is proportional to the sine of the angle between the other two forces. There are three forces P, Q and R acting at a point O which between each of two forces has angle as shown as follow[1].

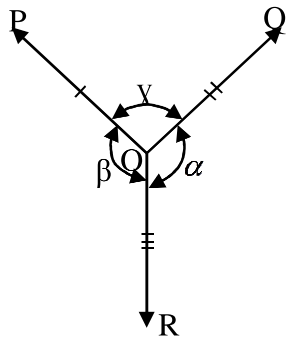


Figure 8 : Lami's theorem

These forces are in equilibrium then according to Lami's theorem that is given as follow.

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma} \qquad \text{Eq (17)}$$

c. Resultant of Any Number of Forces Applied to a Rigid Body at the Same Point

The three-dimensional components and associated quantities of a vector R as shown in figure 9. The unit vector *n* is collinear with R. A unit vector is a vector with the magnitude equal to 1 of sum of unit cordinate vectors and a defined direction[10].

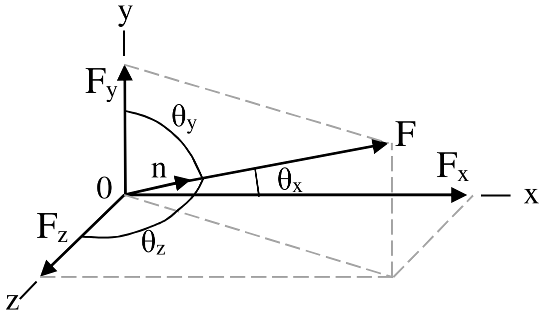


Figure 9 : Three-dimensional components of a vector R

The vector R is written in terms of its scalar components and the unit coordinate vectors as follow.

$$R = F_x i + F_y j + F_z k = Rn \qquad \text{Eq (18)}$$

Where, each of components is defined as below.

$$F_x = R \cos \theta_x \qquad \text{Eq (19)}$$

$$F_y = R \cos \theta_y \qquad \text{Eq (20)}$$

$$F_z = R \cos \theta_z \qquad \text{Eq (21)}$$

And, resultant can be obtained by

$$R = \sqrt{F_x^2 + F_y^2 + F_z^2} \qquad \text{Eq (22)}$$

However, the method is to find unit vector, *n* on the line of points A(x_1, y_1, z_1) and B(x_2, y_2, z_2) can be obtained by

$$n = \frac{vector\ A\ to\ B}{distance\ A\ to\ B} = \frac{d_x i + d_y j + d_z k}{\sqrt{d_x^2 + d_y^2 + d_z^2}} \qquad \text{Eq (23)}$$

Where,

dx = $x_2 - x_1$
dy = $y_2 - y_1$
dz = $z_2 - z_1$

d. Moment of The Force

Moment of the force, or torque is the turning effect of a force on a body. Moment of force can be considered with respect to a point and straight line that is described as follow.

i. The Moment of a Force with Respect to a Point

The tendency of a force to make a rigid body rotate is measured by the moment of that force about an axis. The moment of a force F about an axis through a point O is defined as the product of the magnitude of F times the perpendicular distance d from the line of action of F and the axis O. It is shown in figure 10.

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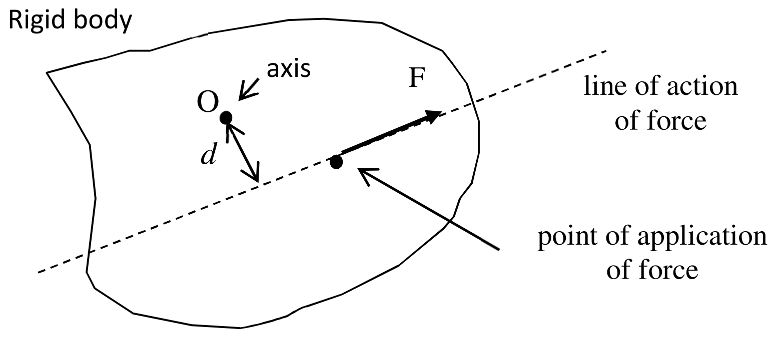

Figure 10 : moment of the force

The equation of this case is defined as a scalar quantity as follow.

$$Mo = F.d \qquad \text{Eq (24)}$$

Where,

Mo = moment about O, lbf.ft (Nm)
F = force, lbf (N)
d = distance, ft (m)

Clockwise moment is reckoned positive and counterclockwise moments negative[7]. When the line of action of a force passes through the axis, the moment is zero, Mo = 0 and the system is in equilibrium. Two forces of equal magnitude and acting along the same line of action have not only the same components F_X, F_Y, but have equal moments about any axis. They are called equivalent forces since they have the same effect on a rigid body.

Moment of force also may be considered by vector quantity. The moment of a bound vector, v about a point A may be illustrated in figure 11[8].

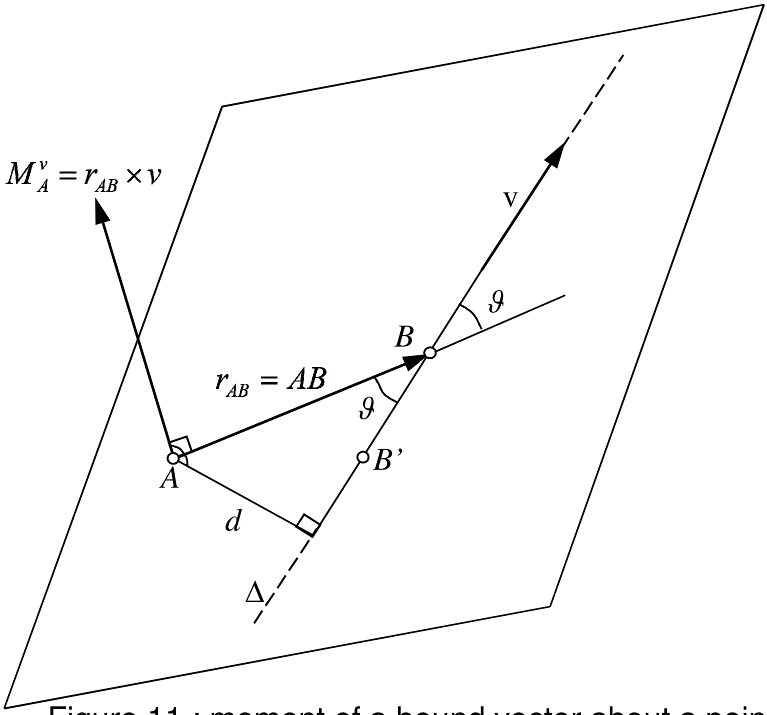

Figure 11 : moment of a bound vector about a point

The vector definition of the moment of this case can be given by

$$M_A^v = AB \times v = r_{AB} \times v \qquad \text{Eq (25)}$$

Where,

r_{AB} = position vector from point A to any point on the line of action of B.

The magnitude of the moment, M_A^v is defined by

$$\left|M_A^v\right| = M_A^v = \left|r_{AB}\right|\left|v\right| \sin\theta \qquad \text{Eq (26)}$$

$$\left|M_A^v\right| = M_A^v = d\left|v\right| \qquad \text{Eq (27)}$$

ii. The Moment of a Force with Respect to a Line

If the force is resolved into components parallel and perpendicular to the given line, the moment of the force with respect to the line is the product of the magnitude of the perpendicular component and the distance from its line of action to the given line. It is common that a body rotates about an axis. In that case the moment of a force about the line is usefully expressed as

$$M_l = n \cdot M_o = n \cdot \left(r \times F\right) = \begin{vmatrix} n_x & n_y & n_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \qquad \text{Eq (28)}$$

Where,

n = a unit vector along the line

r = position vector from point *O* on line to a point on the line of action of F

M_l = projection of M_O on line

e. Couple

A moment is called a couple when two equal in magnitude and opposite forces have parallel lines of action a distance a apart. This is independent of *d* and the resultant force is zero. The only motion that a couple can impart is a rotation. In addition, the couple has no tendency to translate a rigid body. The moment about any point 0 at distance *d* from one of the lines of action is expressed as Eq (29)[7].

$$M = Fd - F(d - a) = Fa \qquad \text{Eq (29)}$$

The moment of a couple about a point is called the torque of the couple, M or T. The moment of the resultant force about any point on the line of action of the given single force must be of the same sense as that of the couple, positive if the moment of the couple is positive conversely. A couple can be illustrated in following figure.

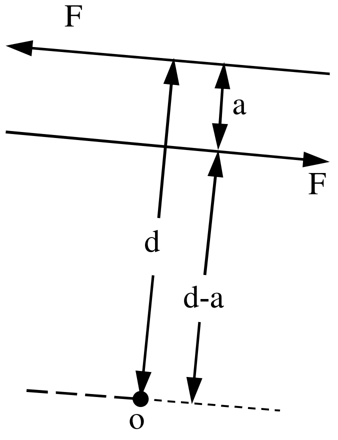

Figure 12 : A couple

f. Equilibrium of Rigid Bodies

A body is in equilibrium when it is stationary or in steady translation relative to an inertial reference frame. In fact, the concept of equilibrium is used for determining unknown forces and moments of forces that act on or within a rigid body or system of rigid bodies. A rigid body is in static equilibrium when the equivalent force-couple system of the external forces acting on it is zero. If the sum of the forces acting on a body is zero and the sum of the moments about one point is zero, then the sum of the moments about every point is zero. In vector notation, this condition is expressed as[8]

$$\sum F = 0 \qquad \text{Eq (30)}$$

$$\sum M_O = \sum (r \times F) = 0 \qquad \text{Eq (31)}$$

In terms of rectangular scalar components, equilibrium can be expressed as

$$\sum F_x = 0 \qquad \sum M_x = 0 \qquad \text{Eq (31)}$$

$$\sum F_y = 0 \qquad \sum M_y = 0 \qquad \text{Eq (32)}$$

$$\sum F_z = 0 \qquad \sum M_z = 0 \qquad \text{Eq (33)}$$

The three couples which may be combined by their moment vectors into a single resultant couple having the moment whose moment vector makes angles. The resultant can be calculated as resultant of any number of forces applied to a rigid body at the same point by replacing force, F with moment, M.

g. Support of Rigid Bodies

According to the number of unknown forces existing, the first step in the solution of problems in statics is the determination of the supporting forces which is defined as the external forces in equilibrium acting upon a body. When the forces are all in either the same or different planes and act at a common point, two or three unknown forces may be determined if their lines of action are known, one if unknown. The following data are required for the complete knowledge of supporting forces: magnitude, direction, and point of application.